

R E C E P T I O N

Addition +

They will begin to relate addition to **combining two groups of objects**, first by **counting all** and then by **counting on** from the largest number.

They will find one more than a given number.

In practical activities and through discussion, they will begin to use the vocabulary involved in addition.

You have four bananas and I have three bananas.



How many bananas altogether?

Subtraction -

In practical activities and through discussion, they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' **using objects** to count 'how many are left' after some have been taken away.

I have six apples. I take two apples away.



How many are left?

Multiplication

In practical activities and through discussion, they will begin to solve problems involving doubling.

Three pencils for you and three pencils for me.



How many pencils altogether?

Division

In practical activities and through discussion they will begin to solve problems involving halving and sharing.

Share the toy cars between two people.



Half of the toy cars for you and half of the toy cars for me.

YEAR ONE

Addition +

Given a number, they must be able to identify one more.

They must read, write and interpret mathematical statements involving addition (+) and the equals (=) sign.

They must be able to add one-digit and two-digit numbers within 20, including zero.

They must be able to solve missing number problems e.g. $10 + _ = 16$

Children should continue to practise counting on from any number e.g. 'Put five in your head and count on four.'

Initially use a **number track** to count on for addition, counting on from the largest number:

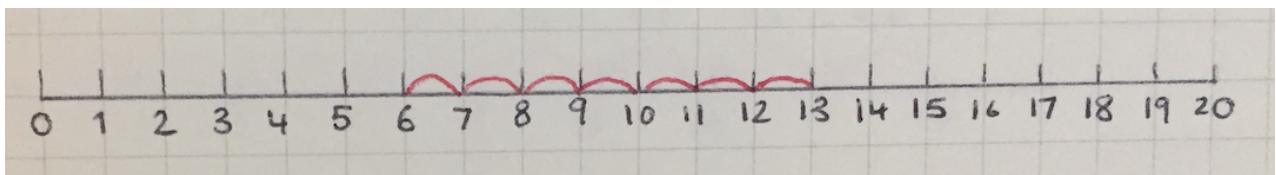


$$5 + 4 = 9$$

'Put your finger on number five. Count on (count forwards) four.'

After confidence has been built with this, progress to a **marked number line**:

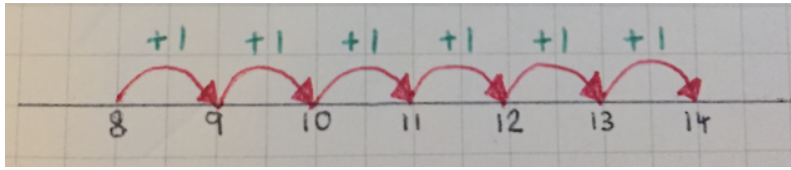
$$6 + 7 = 13$$



Put your finger on number six and count on six.

$$8 + 6 = 14$$

'Put your finger on number eight and count/ jump on six.'



Continue to practise counting on from the largest number for addition with totals within 20.

When children are ready, introduce calculations with totals beyond 20
e.g. $18 + 6 = 24$

Ensure children are confident with using a number track and then a marked number line before moving on to an empty number line (see Y2 guidance).

Subtraction -

Given a number they must be able to identify one less.

They must read, write and interpret mathematical statements involving addition (+) and the equals (=) sign.

They must be able to take away one- digit and two-digit numbers within 20, including zero.

They must be able to solve missing number problems e.g. $10 - _ = 4$

Children should continue to practise counting back from any number e.g. 'Put six in your head and count back four.'

Initially use a **number track** to count back for subtraction, counting back from the largest number:

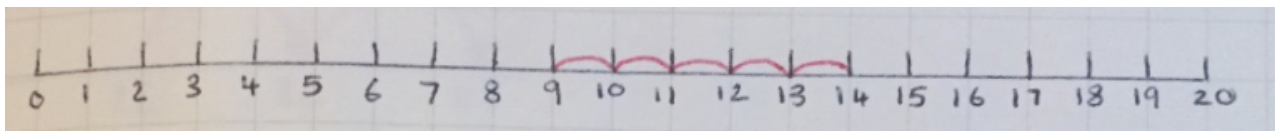


$$9 - 4 = 5$$

'Put your finger on number nine. Count back (count backwards) four.'

Then progress to a **marked number line**:

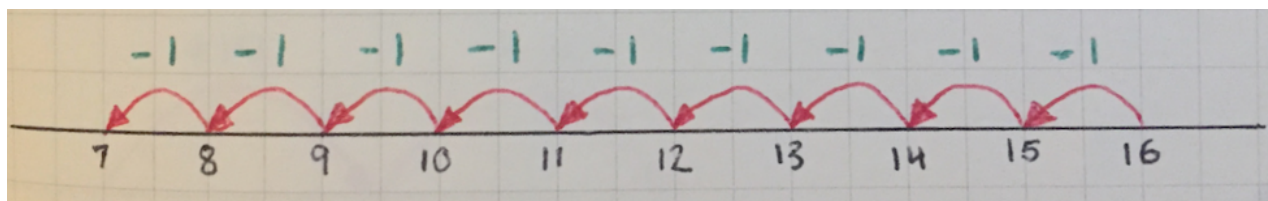
$$14 - 5 = 9$$



Put your finger on number twelve and count back six.

$$16 - 9 = 7$$

'Put your finger on number sixteen and count/ jump back nine.'

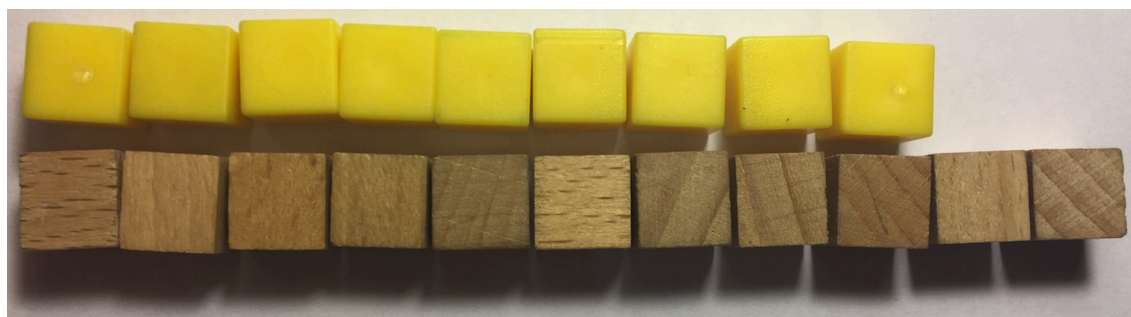


Continue on to find a small difference:

Introduce complementary addition to find differences (only use for small differences). The use of models is extremely important here to understand the idea of 'difference'.

Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

$$11 - 9 = 2$$



Continue to practise counting back in subtractions starting with numbers up to 20.

When children are ready, introduce calculations with starting numbers beyond 20

e.g. $23 + 6 = 17$

Ensure children are confident with using a marked number line before moving on to an empty number line (see Y2 guidance).

Multiplication

Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays

Count in multiples of twos, fives and tens (to the 10th multiple)

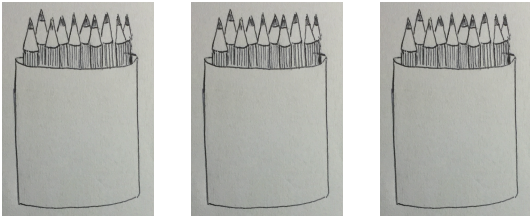
Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts.

Use this to solve **practical problems** that involve combining groups of 2, 5 or 10 e.g. socks, fingers and cubes.



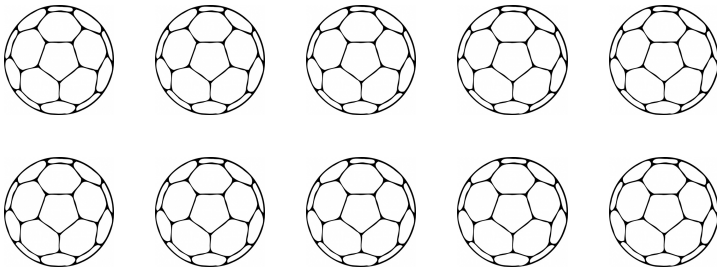
‘Five pairs of socks.

How many socks altogether? 2, 4, 6, 8, 10’



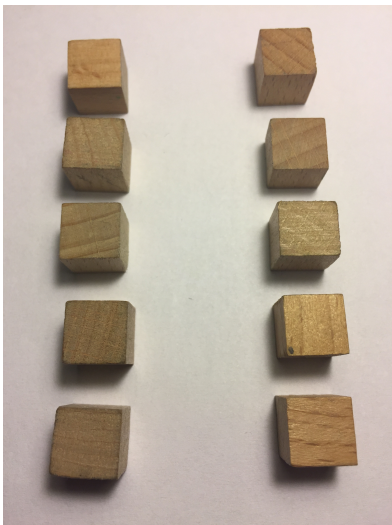
'Three pots of ten crayons. How many crayons altogether? 10, 20, 30'

Use **arrays** to support early multiplication



'Five groups of two footballs. How many footballs altogether? 2, 4, 6, 8, 10'

'Two groups of five footballs. How many footballs altogether? 5, 10'



'2 groups of 5'

'How many altogether?'

' $5 + 5 = 10$ '

Double five is ten

Continue to solve problems **in practical contexts** and develop the language of early multiplication (but **not** the multiplication sign until Y2), with appropriate resources, throughout Y1.

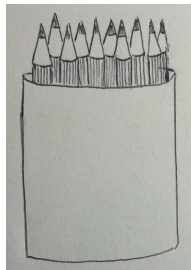
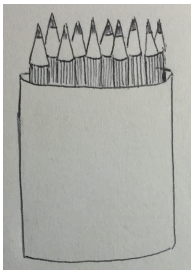
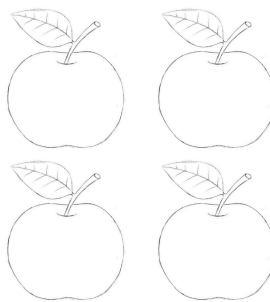
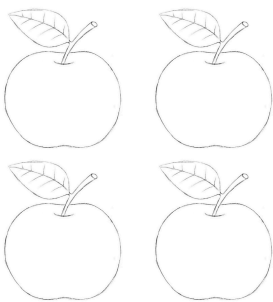
Division

Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays

Count in multiples of twos, fives and tens (to the 10th multiple)

Children will start with practical **sharing** using a variety of resources. They will share objects into **equal groups** in a variety of situations. They will begin to use the vocabulary associated with division in practical contexts.

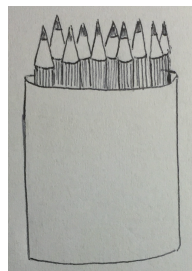
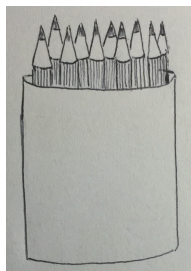
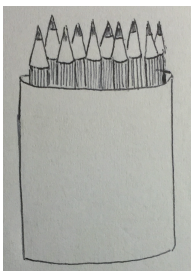
'Share these eight apples equally between two children. How many apples will each child have?'



'Share 20 crayons between 2 pots.'

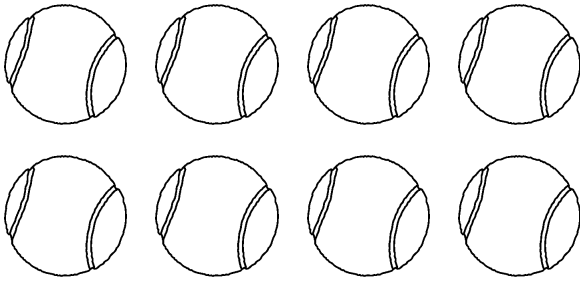
'How many crayons are in each pot?'

Children will move from **sharing** to **grouping** in a practical way

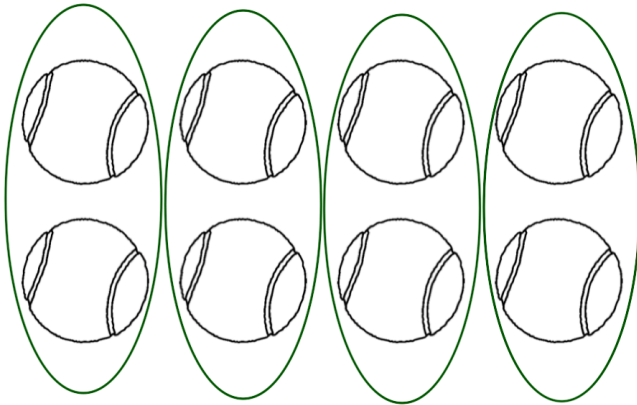


'Put 30 crayons into groups of 10. How many pots do we need?'

Use **arrays** to support early division



'How many tennis balls altogether? How many groups of two?'



'Four groups of two'



'How many groups of 4?'

'8 shared equally between 2 people'

'Half of 8 is four'

Continue to solve problems in **practical contexts** throughout Y1, and develop the language of early division (but **not** the division sign until Y2), with appropriate resources.

YEAR TWO

Addition

Add numbers using concrete objects, pictorial representations, and mentally, including:

- o A two digit number and ones
- o A two digit number and tens
- o Two two-digit numbers
- o Three one-digit numbers (using a mental method)

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.
Consolidate counting on using a **marked number line**.

After this go through the following objectives:

Count on in ones using an **empty number line**, within 100:

$$28 + 6$$

To solve an addition such as this, start by marking the larger number on the left hand side of the empty number line.

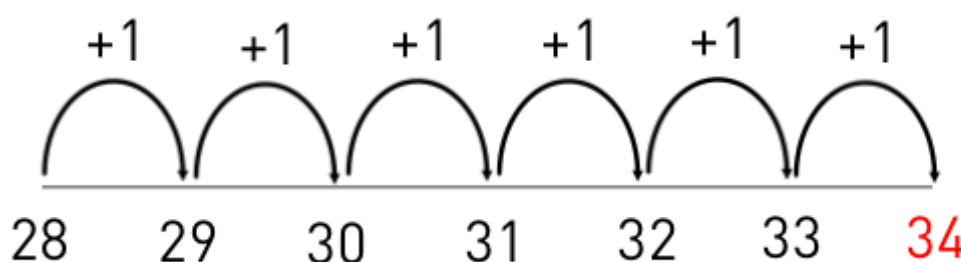
28

Next, add 6 ones, starting like this.



28 29

After you've added six ones it should look like this:



The number
at the end of
your number
line is the
answer:

$$28 + 6 = 34$$

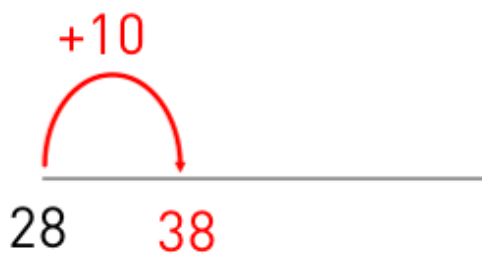
Add tens within 100:

$$28 + 30$$

To solve an addition such as this, start by marking 28 on the left hand side of the empty number line.

28

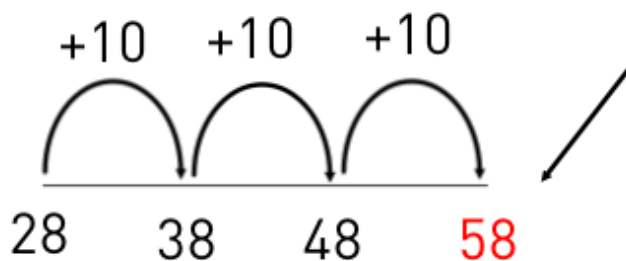
Next, add 3 tens, starting like this.



This is when you should look at a **100 square** to show jumps of tens.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

After you've 3 tens it should look like this:



The number
at the end of
your number
line is the
answer:

$$28 + 30 = 58$$

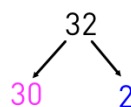
Add 2 two-digit numbers, within 100

$$46 + 32$$

As before put the largest number (46) first on the left:

46

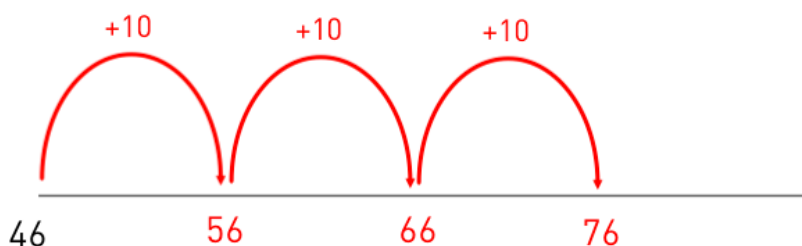
Then partition the smaller number (32) into tens and units:



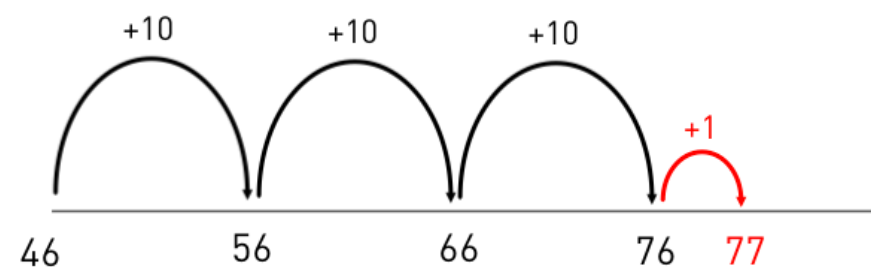
$$32 = 30 + 2$$

To do this addition, we'll count on the tens and then the units: $46 + 30 + 2$.

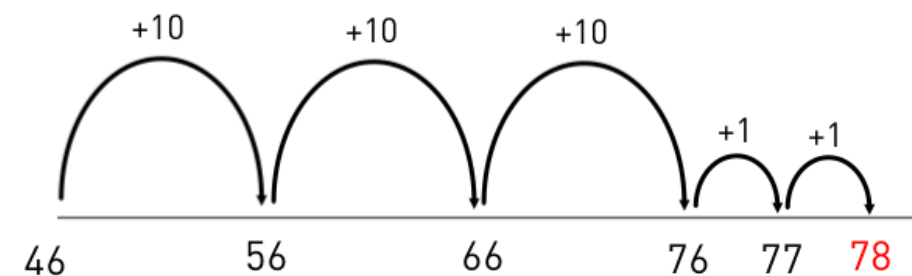
Add the 3 tens first:



Then begin to add the 2 ones like this:



After you've 3 tens and 2 ones, it should look like this:



This is when you should look at a **100 square** to see the jumps of tens and ones.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

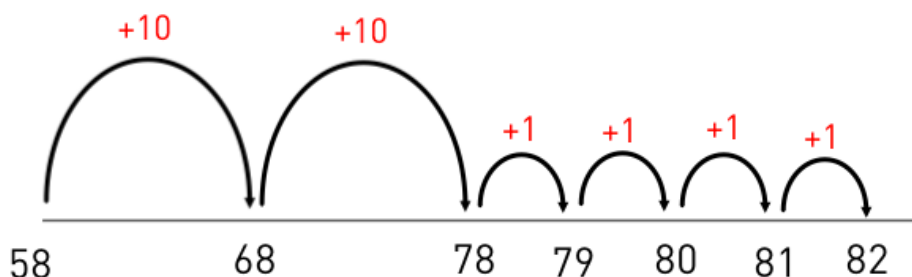
The number at the end of your number line is the answer:

$$46 + 32 = 78$$

Add 2 two-digit numbers within 100 where the units cross a tens barrier.

$$58 + 24$$

To solve this, follow the steps on the previous page and you should record it as follows:



This is where we have crossed a tens barrier: we've gone from counting in the 70s to the 80s. This is a tricky skill to begin with.

Add 2 two-digit numbers within 100 where the units cross a tens barrier, using efficient jumps.

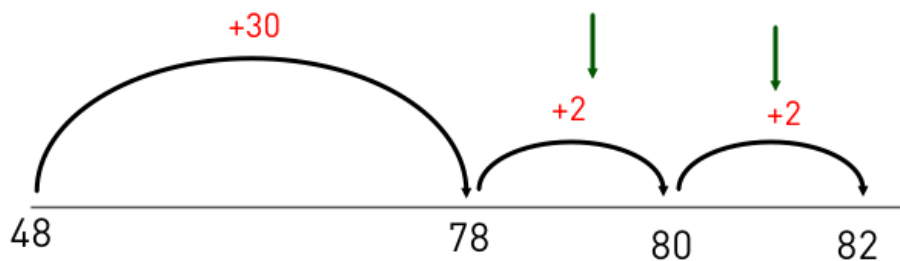
When children are confident with the step before, encourage more efficient jumps. This requires good knowledge of your number bonds and plenty of practice counting in tens:

$$48 + 34$$

Adding the 3 tens in one jump.

What needs to be added to 78 to get to 80?

We need to add another 2 because $4 = 2 + 2$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Remember: for both of these additions, you should have a **100 square** to see the jumps of tens and units.

The **partitioning method** is an alternative method for adding 2 two-digit numbers:

$$43 + 25$$

To partition a two-digit number you must split it into **tens** and **units**. Partition the larger number first starting with its **tens** as follows:

$$\begin{array}{c} 43 + 25 \\ \swarrow \\ 40 \end{array}$$

Then move onto its **units**:

$$\begin{array}{c} 43 + 25 \\ \swarrow \searrow \\ 40 \quad 3 \end{array}$$

Do the same for the smaller number:

$$\begin{array}{c} 43 + 25 \\ \swarrow \searrow \swarrow \searrow \\ 40 \quad 3 \quad 20 \quad 5 \end{array}$$

Once you have partitioned the numbers into tens and ones, add the tens together and then add the ones together.

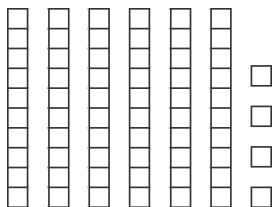
$$40 + 20 = 60$$

$$3 + 5 = 8$$

Recombine the answers to both of these to give the total.

$$60 + 8 = 68$$

Make sure that you can demonstrate what is happening using Dienes (this will improve and support understanding).



Adding 2 two-digit numbers using the **partitioning method** where the units cross a tens barrier:

When children are confident with the step before, move on to calculations that bridge tens:

$$48 + 36 = 40 + 8 + 30 + 6$$

$$40 + 30 = 70$$

$$8 + 6 = 14$$

$$70 + 14 = 84$$

$$48 + 36 = 84$$

This is an alternative way of recording the partitioning method.

Continue to use base ten apparatus to support understanding.

When children are confident, further develop addition of 2 two-digit numbers with totals greater than 100, using a **200 grid** to support (see Y3 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtraction

Subtract numbers using concrete objects, pictorial representations, and mentally, including:

- o A two digit number and ones
- o A two digit number and tens
- o Two two-digit numbers

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Consolidate counting back using a **marked number line**.

Count back using an **empty number line** within 100, in ones.

$$34 - 6 = 28$$

To solve an subtraction like this, start by marking the larger number on the right hand side of the empty number line.



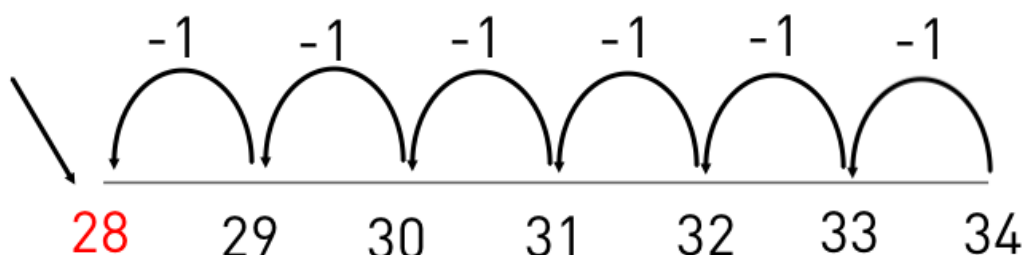
Next, subtract 6 ones, starting like this.



After you've subtracted six ones it should look like this:

The number at this end of your number line is the answer:

$$34 - 6 = 28$$



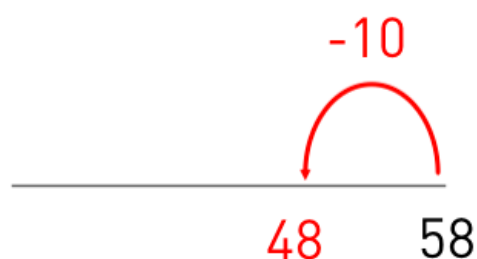
Subtract tens within 100:

$$58 - 30 = 28$$

To solve a subtraction like this, start by marking 58 on the right hand side of the empty number line.



Next, subtract 3 tens, starting like this.



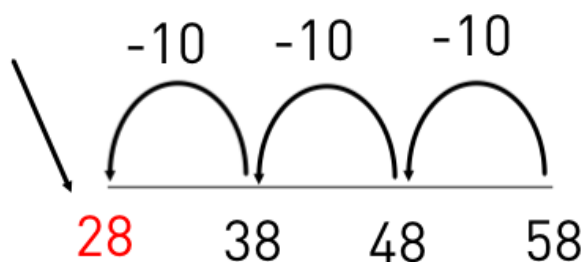
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This is when you should look at a **100 square** to see the jumps of tens.

After you've subtracted 3 tens, it should look like this:

This number at the end of your number line is the answer:

$$58 - 30 = 28$$



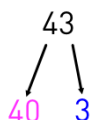
Count back on an **empty number line** to subtract 2 two-digit numbers, within 100:

$$76 - 43 = 33$$

As before put the largest number (76) at the end on the right:



Then partition the smaller number (43) into tens and units:

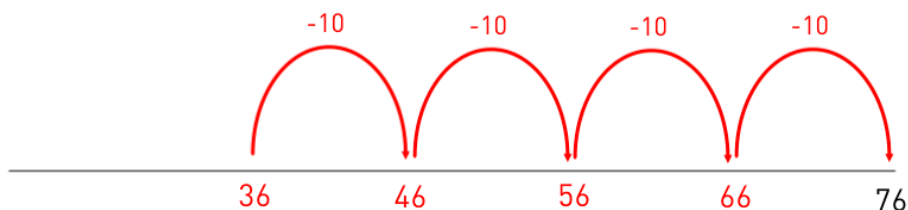


To do this subtraction, we'll count back the **tens** and then the **units**: $76 - 40 - 3$.

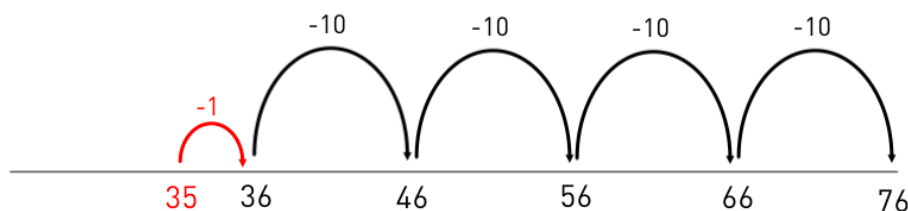
This is when you should look at a **100 square** to see the jumps of tens and ones.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Subtract the 4 **tens** first: .

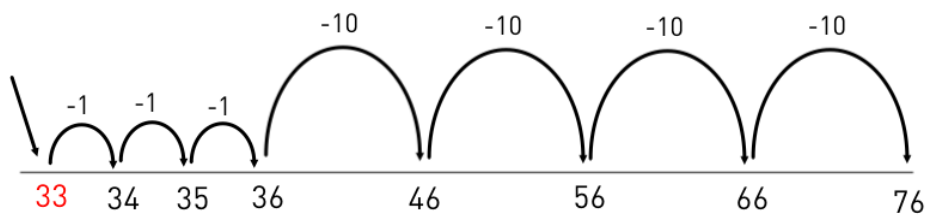


Then subtract the 3 **units**, starting like this:



After you've subtracted 4 tens and 3 units, it should look like this:

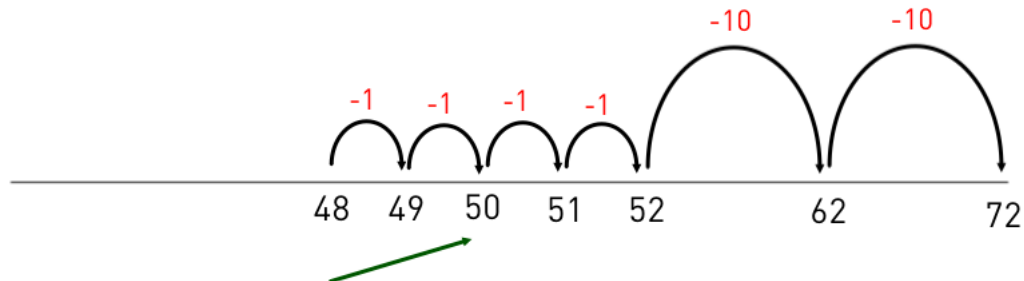
This number at the end of the line is your answer:
 $76 - 43 = 33$



Subtract 2 two-digit numbers within 100 where the units cross a tens barrier.

$$72 - 24 = 48$$

To solve this, follow the steps on the previous page and you should record it as follows:



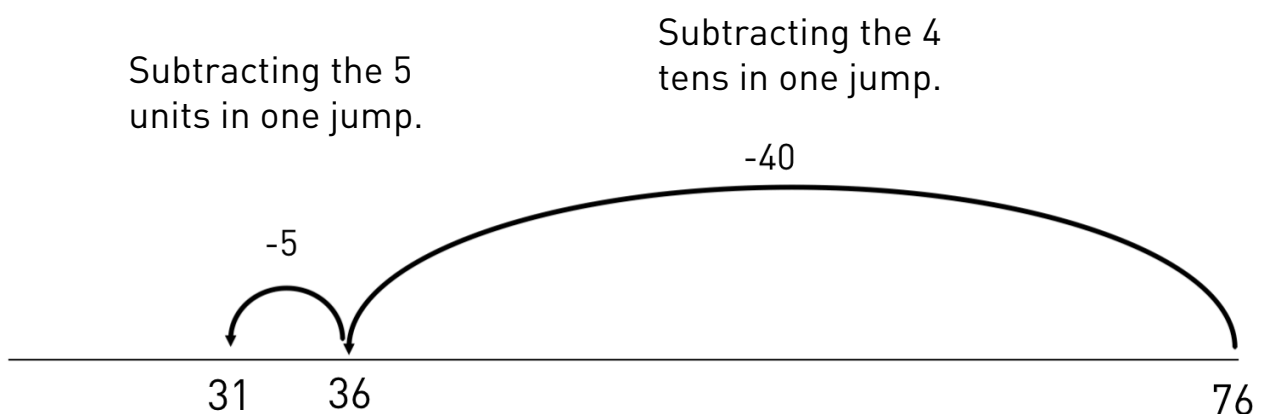
This is where we have crossed a tens barrier: we've gone from counting back in the 50s to the 40s. This is a tricky skill to begin with.

Subtract 2 two-digit numbers within 100 where the units cross a tens barrier, using efficient jumps.

If children are confident, use more efficient jumps:

$$76 - 45 = 31$$

This requires good knowledge of your number bonds and plenty of practice counting backwards in tens:



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Remember: for both of these subtractions, you should have a **100 square** to see the jumps of tens and units.

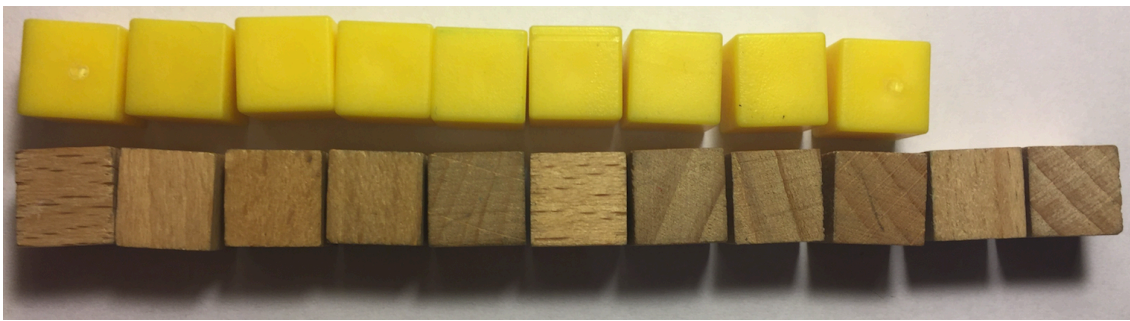
Counting on to find a small difference

Sometimes we actually add to find out the answer to a subtraction. This is called complementary addition and we use it to find differences (we only use this for **small** differences).

The use of models is extremely important here to understand the idea of “difference”. Before looking at how we should show finding the difference in our books, remind yourself of how we find the difference in Year 1.

Count up from the smallest number to the largest to find the difference using resources, e.g. cubes, beads, number tracks/lines:

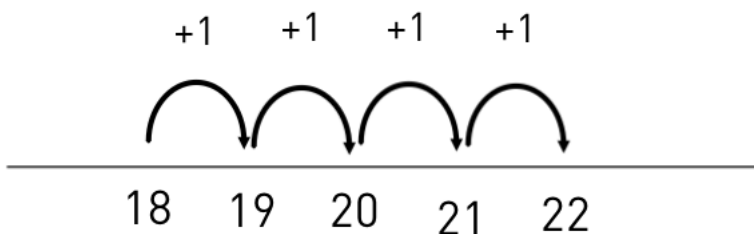
$$11 - 9 = 2$$



We continue this idea but express it using a number line like this:

$$22 - 18$$

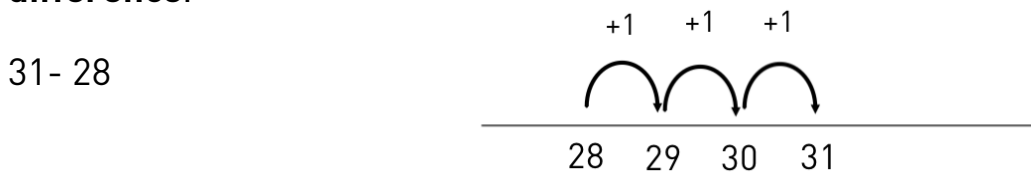
We start with 18 and see what we have to add to get to 22. This is the difference



The difference between 18 and 22 is 4, so $22 - 18 = 4$.

Counting on to find a small difference, **crossing the 10s barrier**

As before, **count up** from the smallest number to the largest to **find the difference**.



The difference between 28 and 31 is 3. So $31 - 28 = 3$

Counting on to find a difference, **using more efficient jumps**.

If children are confident, further develop this method to find the difference, using more efficient jumps:

74 - 58

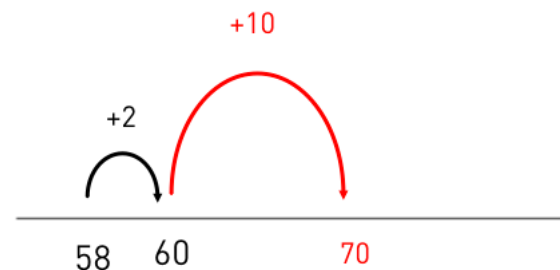
Draw a number line and write the smaller number on the left handside.



We need to add to 58 until we reach 74. To be efficient with this, we first add what is needed to reach the next 10:

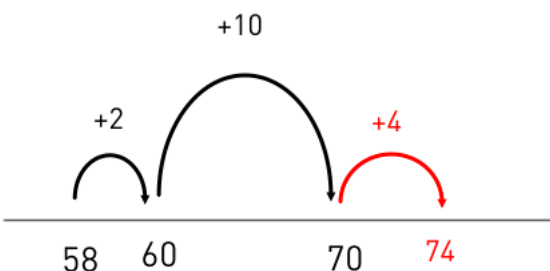


We then add the number of 10s necessary to get as close to our target number as possible: .



In this example, we needed to add one ten, but in other examples, you might need to add more tens when finding the difference .

Finally, we add the remaining units to get to our target number:



We add the jumps together to find the difference: $2 + 10 + 4 = 16$. The difference between 58 and 74 is **16**.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

NB If, at any time, children are making significant errors, return to the previous stage in calculation. And remember, you should use a **hundred square** to help.

Multiplication

Count in steps of 2, 3, 5 and 10 from 0

Recall and use multiplication facts for the 2, 5 and 10 multiplication tables (up to the 12th multiple)

Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (\times) and equals (=) signs

Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

Show that multiplication of two numbers can be done in any order (commutative law)

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the \times sign to record.

Combining Groups (repeated addition) using physical resources:

'5 pots of 10 pencils'

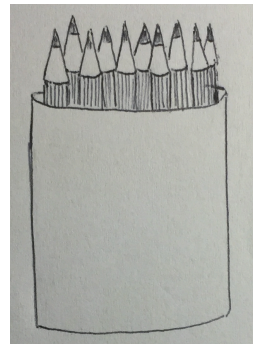
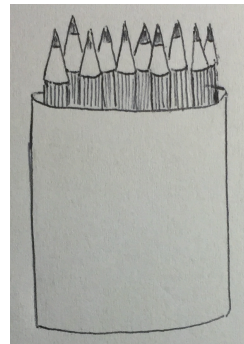
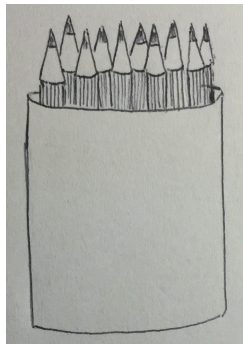
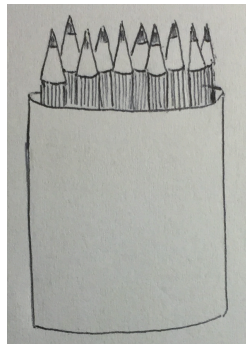
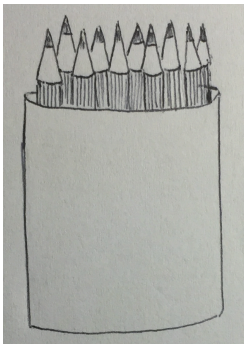
'How many pencils altogether?'

'10, 20, 30, 40, 50'

' $10 + 10 + 10 + 10 + 10 = 50$ '

'5 groups of 10' '5 times ten'

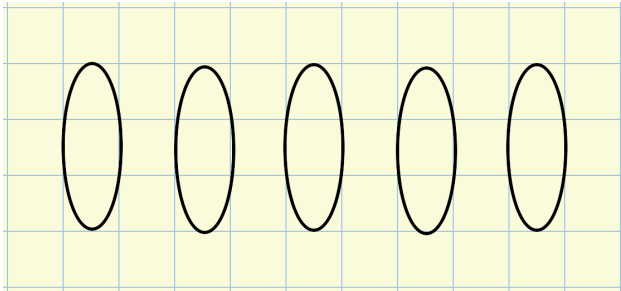
' $5 \times 10 = 50$ '



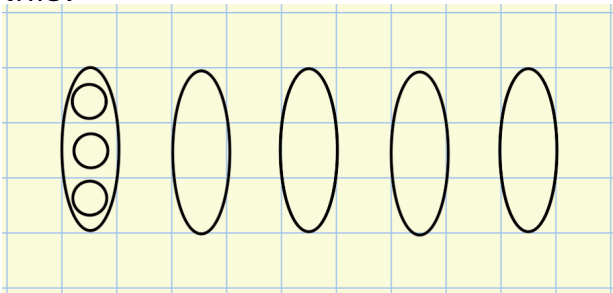
Combining Groups (repeated addition) by drawing diagrams:

'5 groups of 3

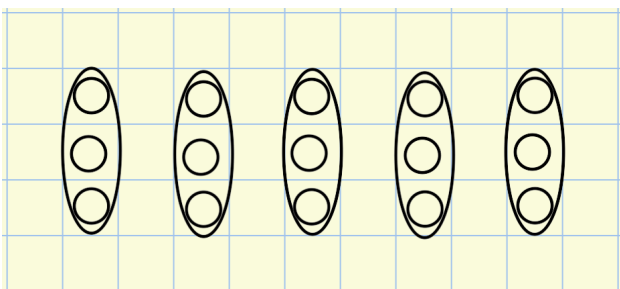
Begin by drawing 5 groups: you might draw 5 circles, ovals or any other shape you like:



Because 5 groups of 3, we need to place 3 in each of our groups, starting like this:



When we've put 3 in each, it should look like this:



We then count each of these 3s:

'3, 6, 9, 12, 15'

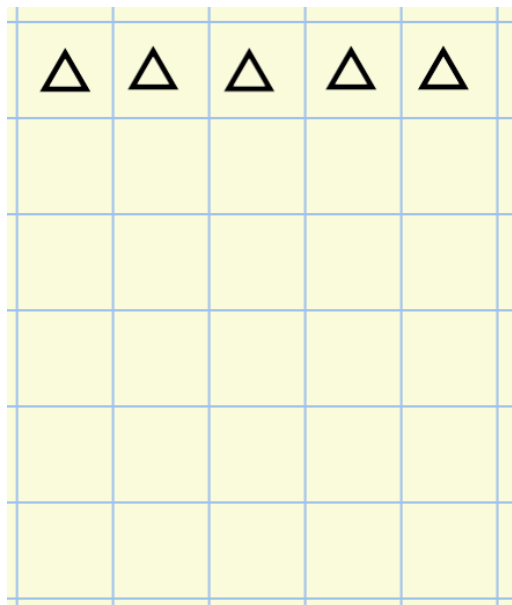
'3 + 3 + 3 + 3 + 3 = 15'

'5 x 3 = 15'

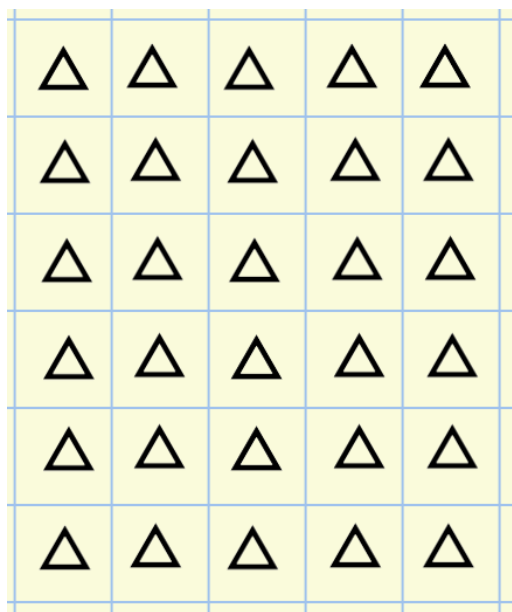
Using arrays to support multiplication:

$$5 \times 6$$

We can draw arrays to demonstrate what 5×6 is. We can start by drawing a row of 5:



We need to draw 6 rows of 5:



Next, we ask: How many stars altogether?

$$'5 + 5 + 5 + 5 + 5 + 5 = 30'$$

'6 rows of 5'

'6 columns of 5'

'5 groups of 6'

$$'6 \times 5 = 30'$$

$$'5 \times 6 = 30'$$

We can see that the multiplication of two numbers can be done in any order: so

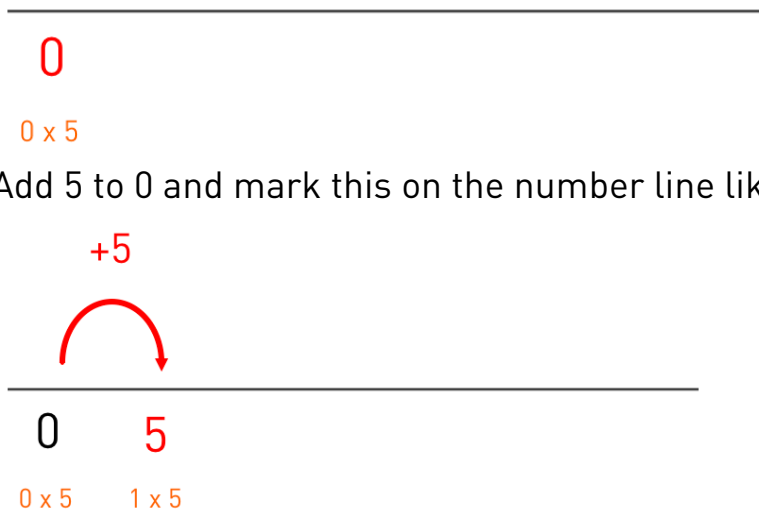
$$5 \times 6 = 6 \times 5$$

Using an empty number line to count on for multiplication:

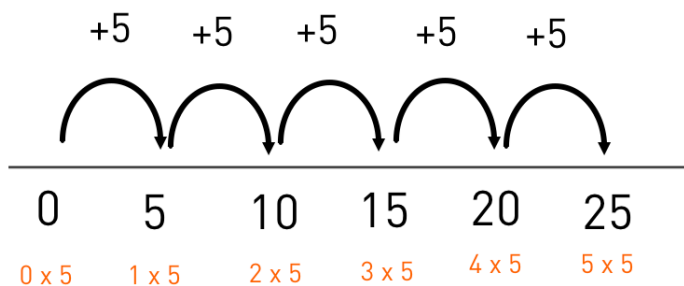
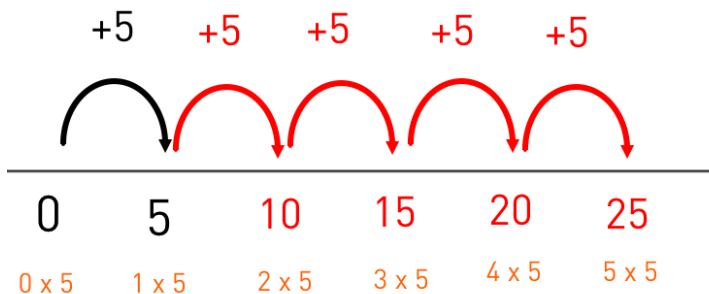
Use an **empty number line** to count on:

$$5 \times 5$$

Begin by marking an empty number line with 0.



Then continue this until you've added 5 lots of 5:



Make the link to repeated addition:

$$5 + 5 + 5 + 5 + 5 = 25$$

Six jumps of five equals 30

$$5 \times 5 = 25$$

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Division

Count in steps of 2, 3, 5 and 10 from 0 (forward and backward)

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables

Calculate mathematical statements for division within the multiplication tables they know and write them using the division (\div) and equals (=) signs

Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts

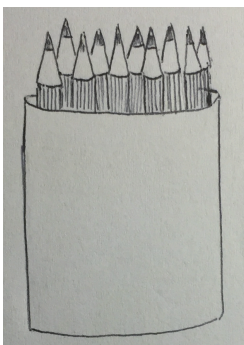
NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the \div **sign** to record, using multiples that they know.

Sharing and grouping using pictures and physical resources to help:

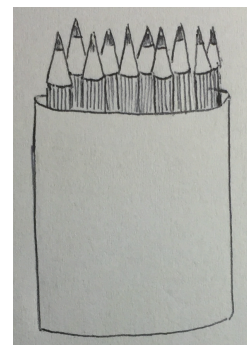
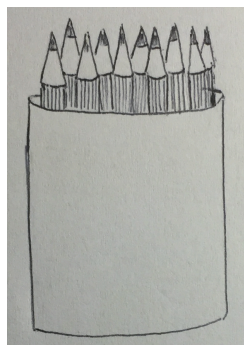
30 crayons shared equally between three pots. How many in each pot?
(Sharing)

$$30 \div 3 = 10$$



We have 30 crayons and put ten crayons in each pot. How many pots do we need?
(Grouping)

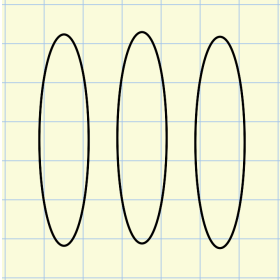
$$30 \div 10 = 3$$



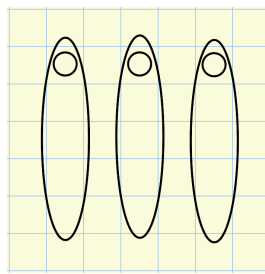
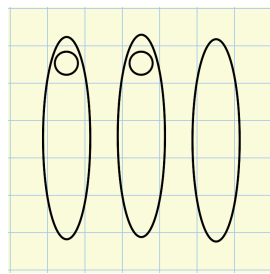
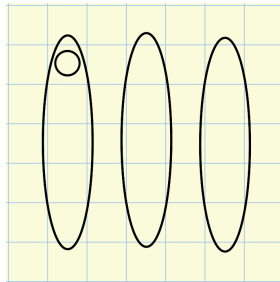
Sharing and grouping using drawing and diagrams to help:

15 toy cars shared equally between three boxes. How many in each box?
(Sharing)

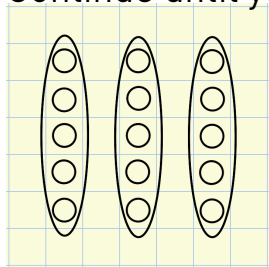
Begin by drawing three circles into which you can share out the 15 objects:



Start sharing out the 15 objects like this:



Continue until you've shared out all 15 objects:



$$15 \div 3 = 5$$

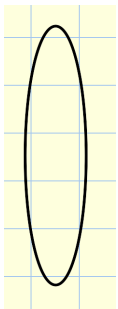
$$15 \text{ divided by } 3 = 5$$

We have shared them out equally and you can see that there are 5 in each.

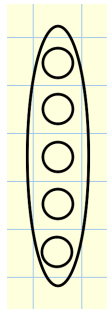
We have 15 toy cars and put five toy cars in each box. How many boxes do we need?

(Grouping)

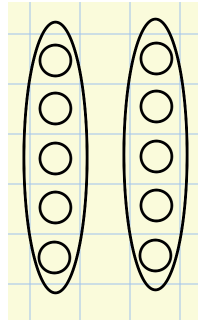
Begin by drawing a shape into which you can place five toy cars:



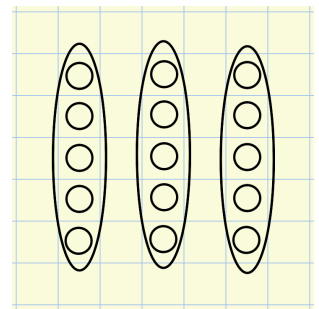
Fill that shape with 5 shapes to represent the toy cars:



Then draw another shape and fill that shape with 5 shapes to represent the toy cars:



Finally draw another shape and fill that shape with 5 shapes to represent the toy cars:



Because we've needed to draw 3 shapes with 5 toy cars in each to get to 15, we know that we need 3 boxes.

$$15 \text{ divided by } 5 = 3$$

$$15 \div 5 = 3$$

Using arrays to support division

$$15 \div 3$$

When solving these, you should know how to draw arrays to help. For $15 \div 5$, you would begin by drawing out a row of 5 since this is the number your dividing 15 by:

△	△	△	△	△

You start by drawing a row of 5 triangles or circles (whatever you prefer). Then you start a new row of 5 like this.

△	△	△	△	△
△	△	△	△	△

Then you start a new row of 5 like this.

△	△	△	△	△
△	△	△	△	△
△	△	△	△	△

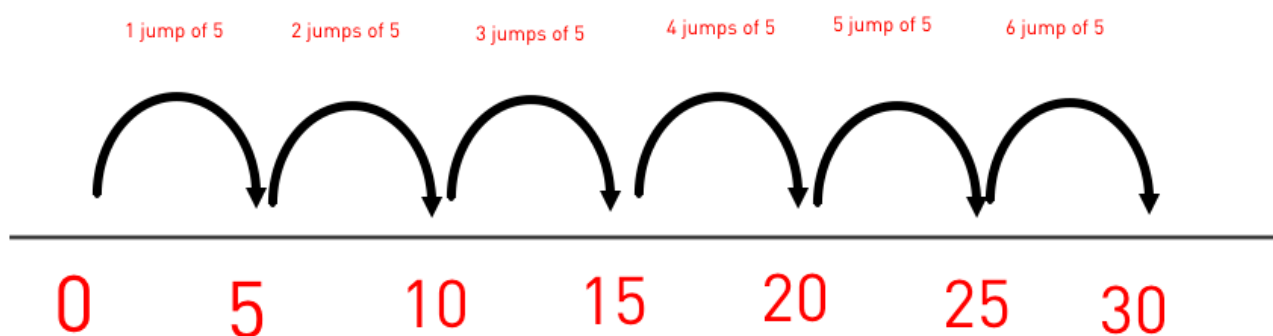
When you have reached the number you're dividing 5 by (15) you should stop.

Count how many rows you have: 1, 2, 3. 3 rows. This means that $15 \div 5 = 3$

Use an **empty number** line to count forwards to divide, making the link with multiplication:

$$30 \div 5$$

‘How many jumps of five make thirty?’



By drawing out the jumps of five, we can see that there are 6 of them.

So $30 \div 5 = 6$

Using an empty number line to count back for division using repeated subtraction:

Use an **empty number line** to count back:

$$30 \div 5$$

‘How many groups of five?’

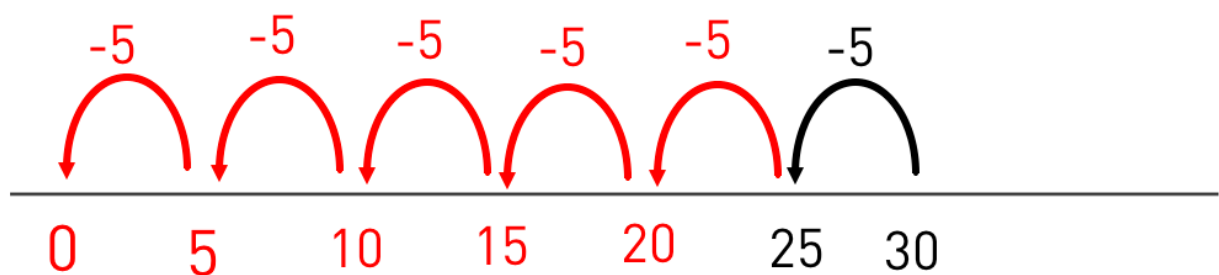
Begin by marking an empty number line with 30, the number you're dividing.



Subtract 5 from 30 on the number line like this:



Then continue subtracting 5s until you get to 0:



How many times did we subtract 5 from 30 until we got to 0?

6

So $30 \div 5 = 6$

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

YEAR THREE

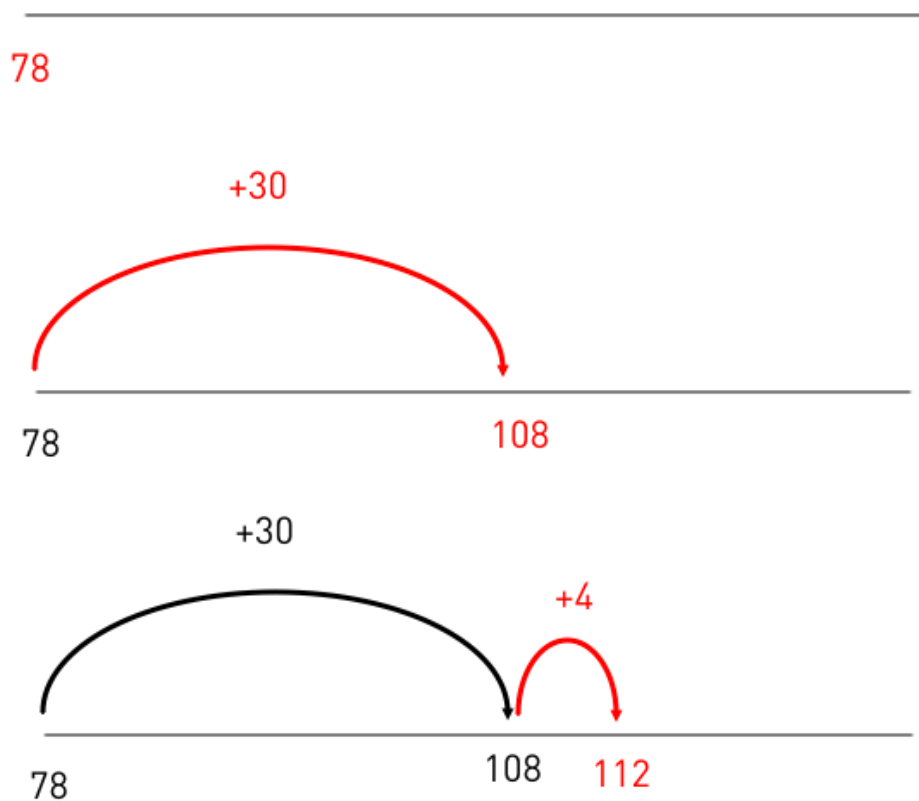
Addition - Year Three

Add numbers with up to three digits, using formal written method of columnar addition

NB: Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the use of the empty number line with calculations that bridge 100:

$$78 + 34$$



Further develop the partitioning method with calculations that bridge 100:

$$85 + 37 = 80 + 5 + 30 + 7$$

$$80 + 30 = 110$$

$$5 + 7 = 12$$

$$110 + 12 = 122$$

$$85 + 37 = 122$$

Consider the use of base ten apparatus (e.g. Dienes) to support understanding.

The partitioning method can also be used with three-digit numbers.

Use a 200 grid to support counting on in tens when bridging 100

Further develop with addition of a three-digit and a two -digit number:

$$125 + 56 = 181$$

125

+50



125

175

+50



+6



125

175

181

Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it is not been necessary to bridge across the tens or hundreds:

6	3	+	3	2
---	---	---	---	---

6	3	+	3	2
↓	↓		↓	↓
60	3		30	2

Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together.

Recombine to give the total.

	6	0	+	3			
+	3	0	+	2			
<hr/>							
	9	0	+	5	=	9	5

Then refine the recording...

		6	3				
+		3	2				
<hr/>				5	(3 + 2)		
+		9	0	(60 + 30)			
<hr/>				9	5		

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

Use base ten apparatus to support understanding.

This will lead into the formal written method...

		6	3
+		3	2
		9	5

Use the language of place value to ensure understanding:
'Three add two equals five. Write five in the ones/units column.

60 add 30 equals 90. Write 9 (90) in the tens column.

The total is 95.

NB Informal/mental methods, such as counting on an empty number line or partitioning would be more appropriate for numbers of this size, but use two-digit numbers, initially, when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an expanded method initially:

	6	8	+	2	4		
	6	0	+	8			
+	2	0	+	4			
	8	0	+	12	=	9	2

Partition the numbers into tens and ones/units.
Add the tens together and then add the ones/units together.

Recombine to give the total (92).

Then refine the recording...

			6	8					
+			2	4					
			1	2				(8 + 4)	
+			8	0				(6 0 + 2 0)	
			9	2					

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

When children are ready, introduce the formal written method, where it is necessary to 'carry' ten from the units to the tens column:

			6	8
+			2	4
			9	2
			1	

Use the language of place value to ensure understanding:
 'Eight add four equals 12. Write two in the units column and 'carry' one ten (10) across into the tens column.
 60 add 20 and the ten that we 'carried' equals 90.
 Write 9 (90) in the tens column. 92 is the answer'.

The digit that has been 'carried' should be recorded under the line in the correct column.

When children are confident, extend with examples where it is necessary to bridge across the tens and the hundreds, returning to an expanded method, if necessary:

			7	0	+	6			
+			4	0	+	7			
			1	1	0	+	13	=	1 2 3

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together.
 Recombine to give the answer.'

Then...

			7	6					
+			4	7					
			1	3				(6 + 7)	
+		1	1	0				(7 0 + 4 0)	
		1	2	3					

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

When children are ready introduce the formal written method, where it is necessary to 'carry' across the columns, including calculations with a total greater than 100:

		7	6
+		4	7
	1	2	3
	1	1	

Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one (10) across into the tens column.

40 add 70 and the ten that we 'carried' equals 120.

Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).

The total is 123'.

The digits that have been 'carried' should be recorded under the line in the correct column.

When children are confident, further develop with the addition of 2 three-digit numbers:

	2	4	3
+	1	7	8
	4	2	1
	1	1	

Continue to use base ten apparatus to support, if necessary.

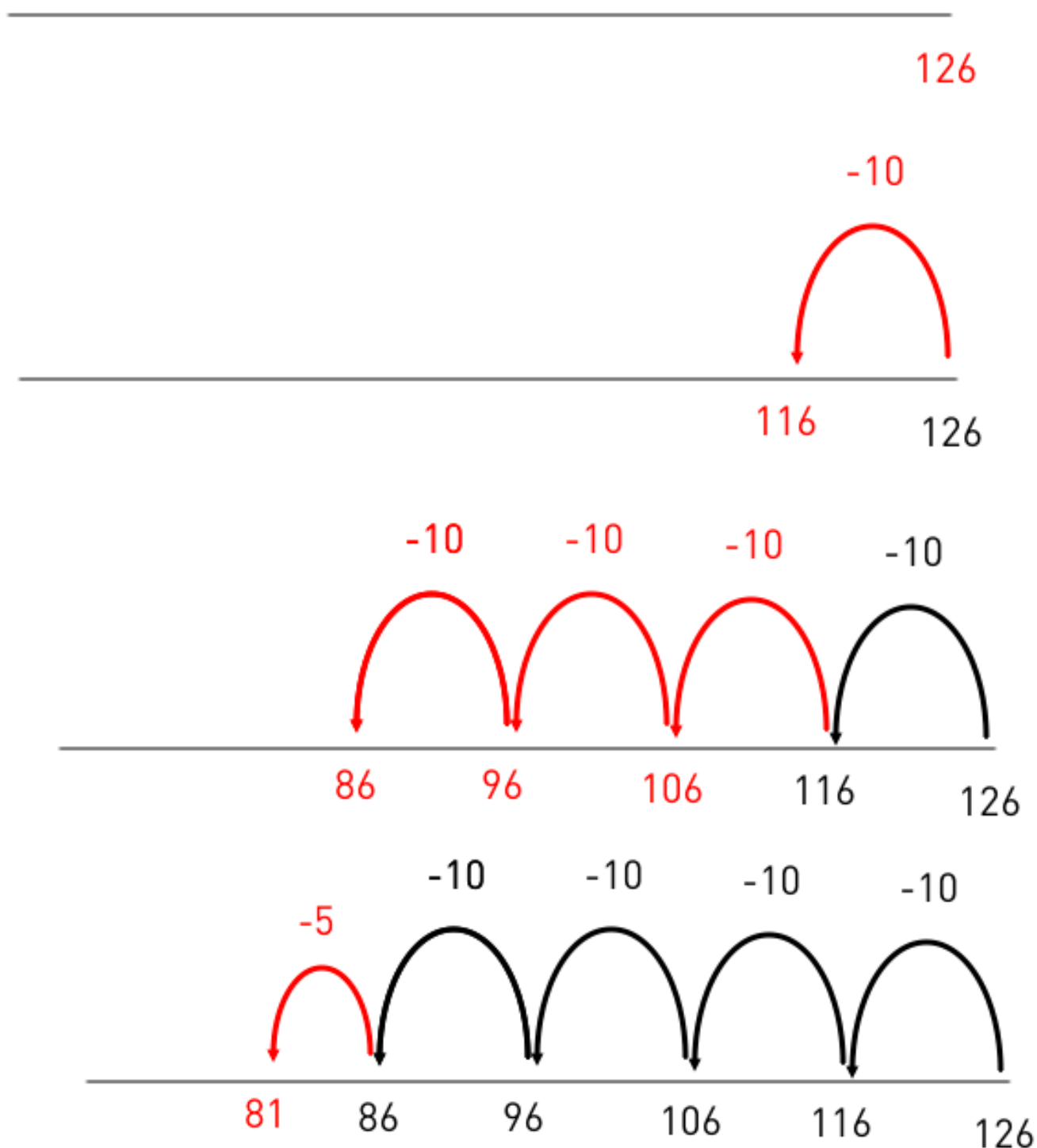
Subtraction - Year Three

Subtract numbers with up to three digits, using formal written method of columnar subtraction

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

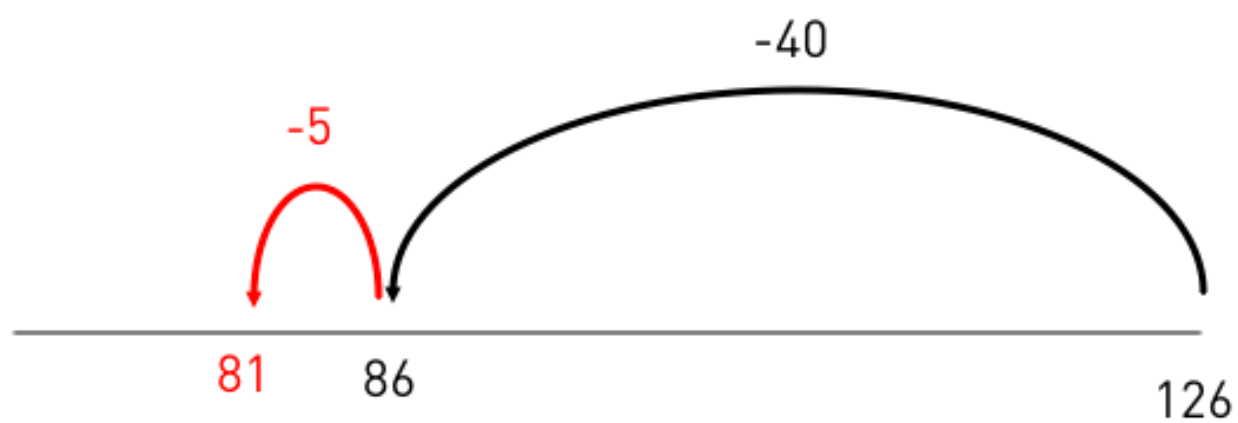
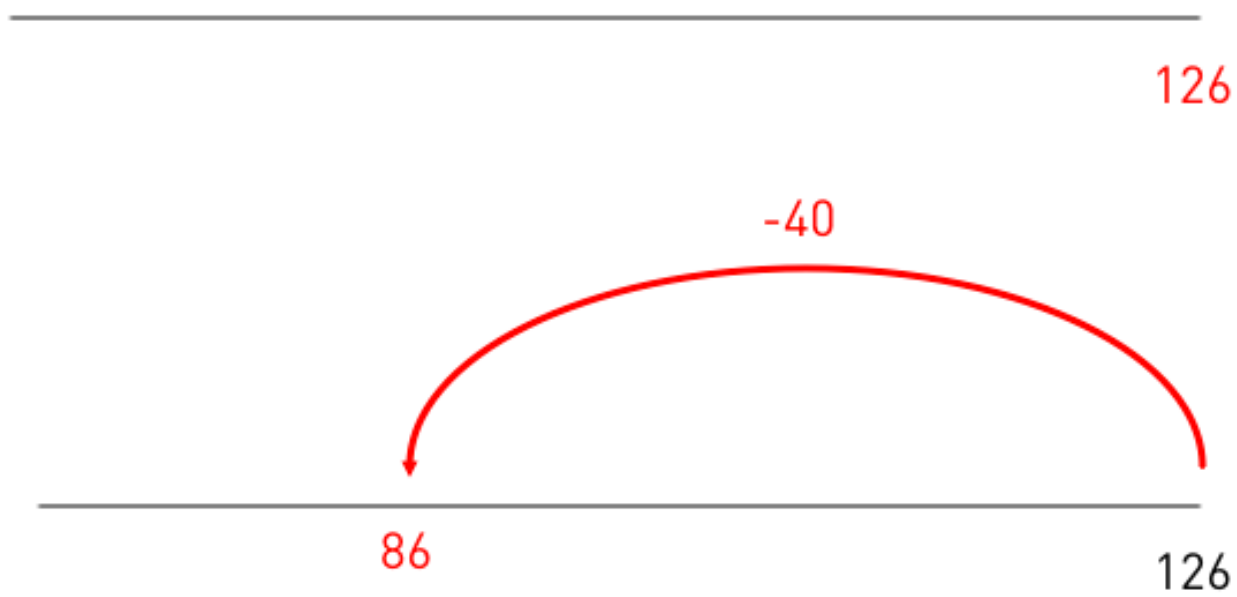
Further develop the use of the empty number line with calculations that bridge 100:

$$126 - 45$$



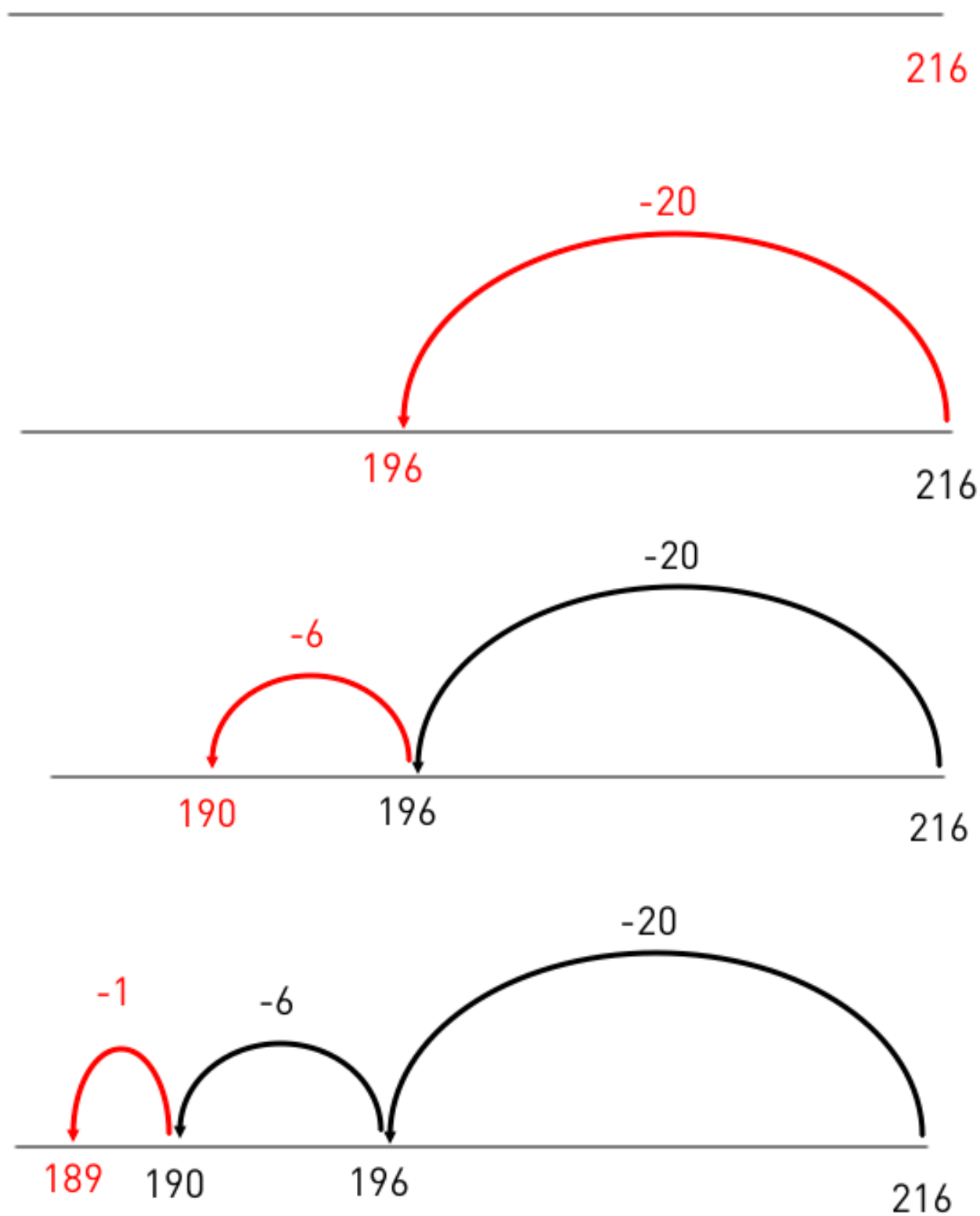
Use a 200 grid to support counting back in tens and bridging 100

Then use more efficient jumps:



Extend with larger numbers by counting back...

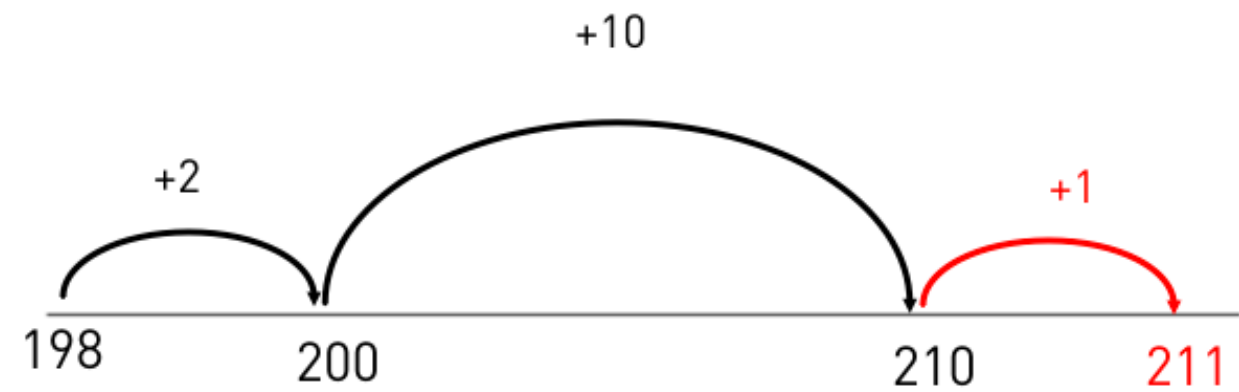
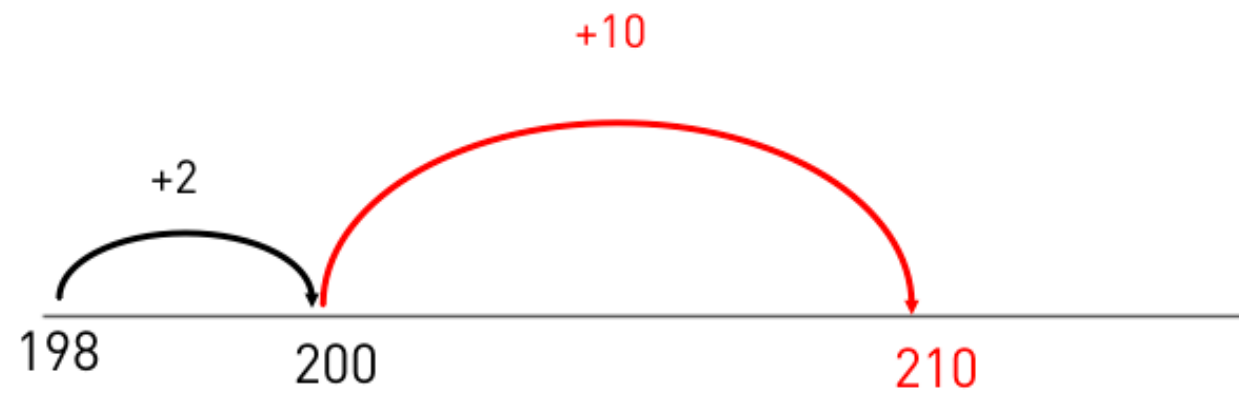
$$216 - 27 = 189$$



...and by counting on to find the difference (small difference):

$$211 - 198 = 13$$

198



‘The difference between 198 and 211 is 13.’

Introduce the expanded written method with the calculation presented both horizontally and vertically (in columns).

Use two-digit numbers when introducing this method, initially:

	7	0	+	8			
-	2	0	+	3			
	5	0	+	5	=	5	5

Partition numbers into tens and ones/units. Subtract the ones, and then subtract the tens. Recombine to give the answer.'

NB In this example exchange is not required.

You might replace the + sign with the word 'and' to avoid confusion.

This will lead into the formal written method, where no exchange is required:

		7	8	
-		2	3	
		5	5	

Use the language of place value to ensure understanding:

'Eight subtract three is five, seventy subtract twenty is fifty. The answer is fifty five'

NB An empty number line would be an appropriate method for this calculation but use two-digit numbers to illustrate the formal written method initially.

Use the expanded written method where exchange is required:

73 is partitioned into 60+13 in order to calculate 73-27

	7	0	+	3				6	0	+	13			
-	2	0	+	7	becomes			-	2	0	+	7		
									4	0	+	6	=	4 6

	6	0		13			
	7	0	+	3			
-	2	0	+	7			
	4	0	+	6	=	4	6

NB children will need to practise partitioning numbers in this way.

Base- ten apparatus (e.g. Dienes) could be used to support this.

When children are confident with the expanded method introduce the formal written method, involving decomposition/exchange:

		6	13
		7	3
-		2	7
		4	6

Use the language of place value to ensure understanding.

'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

Use base ten apparatus to support understanding.

If children are confident, extend the use of the formal written method with numbers over 100, returning to the expanded method first, if necessary.

$$235 - 127 = 108$$

		2	15
	2	3	5
-	1	2	7
	1	0	8

Use the language of place value to ensure understanding.

In this example it has only been necessary to exchange from the tens column.

Use base ten apparatus to support understanding.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication - Year Three

Recall and use multiplication facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods and progressing to a formal written method

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use number lines and arrays to support multiplication, as appropriate.

△	△	△	△
△	△	△	△
△	△	△	△
△	△	△	△
△	△	△	△

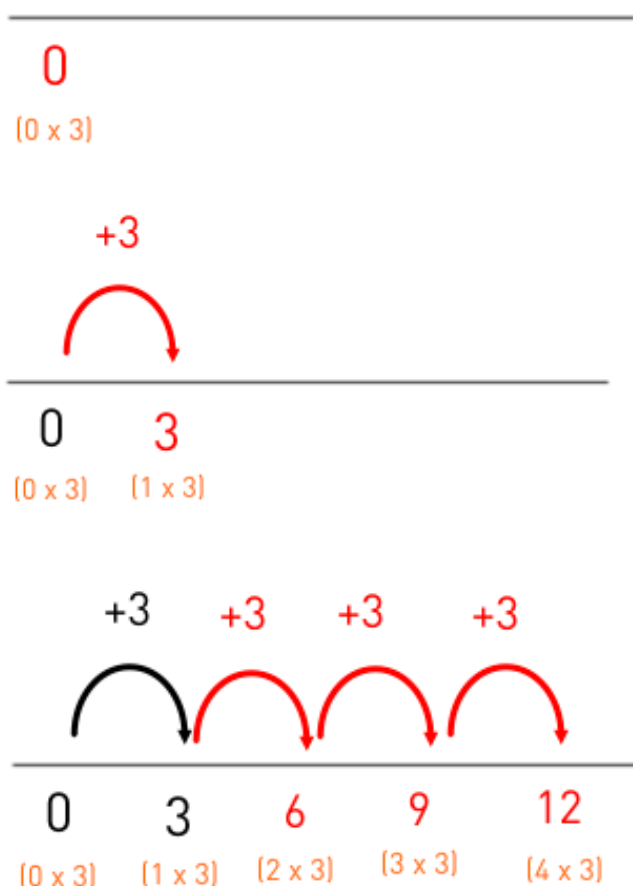
$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

Use an empty number line to count on:

$$4 \times 3 = 12$$

'0, 3, 6, 9, 12'



Include an addition symbol when adding partial products.

Formal short multiplication:

		1	3
×			8
	1	0	4
		2	

Ensure that the digit 'carried over' is written under the line in the correct column.

Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout Y3 using teen-numbers multiplied by a one-digit number.

If children are confident progress to multiplying other two-digit numbers by a one-digit number e.g. 24×3 ; 35×5 (see Y4 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Three – Division

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables (continue to practise the 2, 5 and 10 multiplication tables)

Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the \div sign to record, using multiples that they know, as appropriate (see Y2 guidance).

△	△	△	△
△	△	△	△
△	△	△	△
△	△	△	△
△	△	△	△

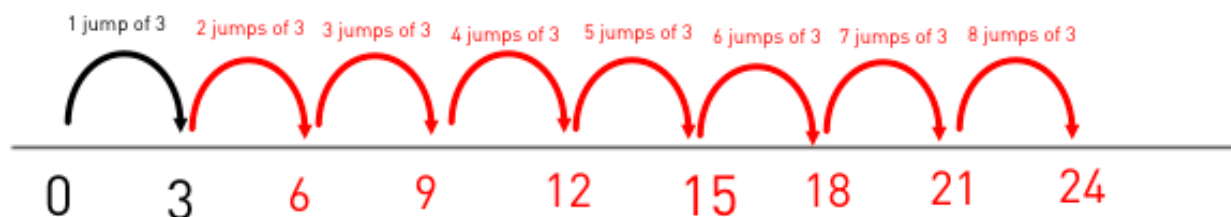
$$20 \div 5 = 4$$

$$20 \div 4 = 5$$

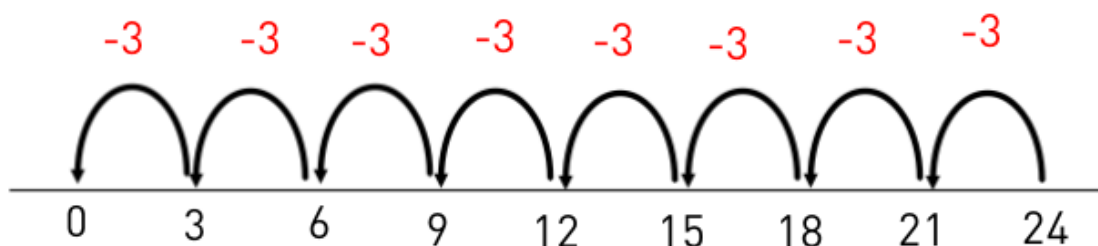
Use an empty number line to count forwards...

$$24 \div 3 = 8$$

'How many threes are there in 24?'



...also jump back from 24 to make the link with repeated subtraction.



Introduce the formal layout using multiplication/division facts that the children know:

$$24 \div 3 = 8$$

This can also be recorded as...

		8
3	24	

'Twenty four divided by three equals eight.'

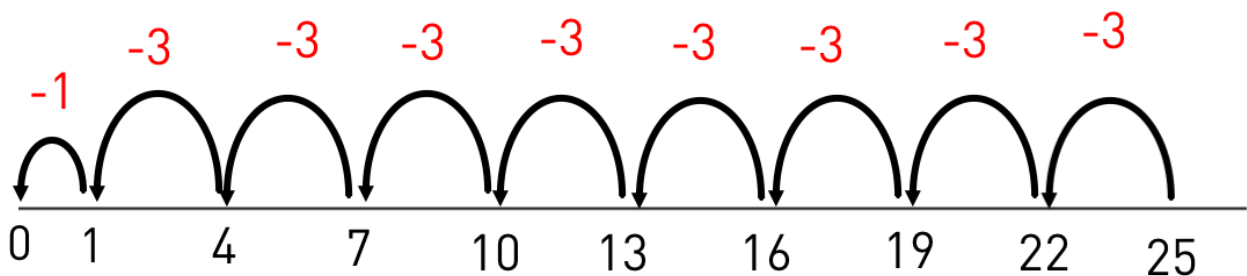
'How many threes are there in twenty four?'

Begin to determine remainders, using known facts e.g. recognise that $13 \div 4$ will have a remainder of 1; $13 \div 4 = 3 \text{ r } 1$

NB Remainders are not specifically referred to until Y5 in the National Curriculum. However, as an understanding of remainders is required for children working at 'greater depth' in Y2, this may be an appropriate point to introduce them.

$$25 \div 3 = 8 \text{ r } 1$$

Eight jumps of three and one left over.'



Alternatively you could jump forwards in multiples of three from zero to twenty four ('and one more makes 25')

		8	r	1
3	25			

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

When children are confident, introduce addition of a four-digit number and a three-digit number:

	1	8	4	5
+		5	2	6
	2	3	7	1
	1		1	

Continue to develop with addition of 2 four-digit numbers and with decimal numbers (in the context of money or measurement).

		4	5	•	6	5
+		2	8	•	5	0
		7	4	•	1	5
		1	1			

Ensure that the decimal points line up.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate

Continue to teach the use of empty number lines with three and four digit numbers, as appropriate.

$$258 - 73 = 185$$
[illegible]

Children will need to practise partitioning in a variety of ways.

This leads to the formal written method, involving exchanging...

		1	15	
		2	5	8
-			7	3
		1	8	5

In this example it has been necessary to exchange from the hundreds column.

Further develop by subtracting a three-digit number from a three-digit number:

$$637 - 252 = 385$$

		6	0	0	+	3	0	+	7				
-		2	0	0	+	5	0	+	2				
<hr/>													
		5	0	0	+	1	3	0	+	7			
-		2	0	0	+	5	0	+	2				
<hr/>													
		3	0	0	+	8	0	+	5	=	3	8	5

Ensure that children are confident in partitioning numbers in this way.

This leads to a formal written method:

		5	13	
		6	3	7
-		2	5	2
		3	8	5

Use the language of place value to ensure understanding and use base-ten apparatus, if necessary.

When children are confident, develop further with four- digit numbers and decimal numbers (in the context of money and measures).

$$3,625 - 1,219 = 2,406$$

			1	15
	3	6	2	5
-	1	2	1	9
	2	4	0	6

$$£56.75 - £34.80 = £21.95$$

			5	17	
		5	6	7	5
-		3	4	8	0
		2	1	9	5

Ensure that the decimal points line up.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Multiplication- Year Four

Recall multiplication facts for multiplication tables up to 12 x12

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the grid method for two-digit numbers multiplied by a one- digit number.

$$36 \times 4 = 144$$

×	3	0	6
4	1	2	0
	2	4	

$$120 + 24 = 144 \text{ (add the partial products)}$$

Expanded short multiplication (two-digit number by a one-digit number):

$$36 \times 4 = 144$$

	3	0	+	6						
×				4						
			2	4			(6 × 4)			
+		1	2	0			(30 × 4)			
		1	4	4						

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication:

			3	6						
×				4						
			2	4			(6 × 4)			
+		1	2	0			(30 × 4)			
		1	4	4						

Short multiplication (formal method) of a two-digit number multiplied by a one-digit number:

$$36 \times 4 = 144$$

	3	6
×		4
<hr/>		
1	4	4
<hr/>		
	2	

Use the language of place value to ensure understanding.

Ensure that the digit 'carried over' is written under the line in the correct column.

Continue to practise the formal method of short multiplication of a two-digit number by a one-digit number throughout Y4.

When children are confident, continue to develop short multiplication with three-digit numbers multiplied by a one-digit number.

If necessary, return to the grid method and/or expanded method first:

$$127 \times 6 = 762$$

×	100	20	7
6	600	120	42

$$600 + 120 + 42 = 762 \text{ (add the partial products)}$$

This leads to the expanded short multiplication method:

$$127 \times 6 = 762$$

	1	2	7					
×				6				
<hr/>								
			4	2		(7 × 6)		
		1	2	0		(20 × 6)		
+		6	0	0		(100 × 6)		
<hr/>								
		7	6	2				

Short multiplication (formal method) of a three-digit number multiplied by a one-digit number:

	1	2	7
×			6
	7	6	2
	1	4	

Use the language of place value to ensure understanding.

Ensure that the digits 'carried over' are written under the line in the correct

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Four- Division

Recall multiplication and division facts for multiplication tables up to 12 x 12

Use place value, known and derived facts to divide mentally

Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

$$32 \div 8 = 4$$

Continue using the formal written layout for division using known multiplication tables:

		4
8	3	2

'How many eights are there in thirty two?'

Continue using the formal written layout, including examples with remainders:

$$33 \div 8 = 4 \text{ r}1$$

		4	r	1
8	3	3		

This could be modelled using an empty number line to ensure understanding, if needed (See Y3 guidance).

Division using partitioning (two- digit numbers divided by a one –digit number):

$$65 \div 5 = 13$$

$$65 = 50 + 15 \quad \text{Partition 65 into 50 and 15}$$

$$50 \div 5 = 10$$

$$15 \div 5 = 3$$

$$10 + 3 = 13$$

NB Children will need to practise partitioning in a variety of ways.

$$98 \div 7 = 14$$

$$98 = 70 + 28 \quad \text{Partition 98 into 70 and 28}$$

$$70 \div 7 = 10$$

$$28 \div 7 = 4$$

$$10 + 4 = 14$$

This could be modelled on an empty number line to further develop understand.

Record the partitioning method using the formal layout

$$98 \div 7 = 14$$

	1	0	+	4	=	1	4
7	7	0	+	2	8		

'We have partitioned 98 into 70 and 28
(90 = 70 + 28).

Seven 'goes into' 70 ten times and seven 'goes
into' 28 four times.

Ten add four equals 14'

This will lead into the formal written method of
short division:

$$98 \div 7 = 14$$

	1	4
7	9	28

Use the vocabulary of place value to ensure
understanding and make the link to partitioning.

Continue to practise the formal method of short division throughout Y4.

If children are confident develop further, by dividing three-digit numbers by a one- digit number using the formal method of short division (first without remainders and then with remainders).

$$132 \div 6 = 22$$

		2	2
6	1	3	2

Use the language of place value to ensure understanding.

Make the link to the partitioning method.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

YEAR FIVE

Addition - Year Five

Add whole numbers with more than 4 digits, including using a formal written method (columnar addition)

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of empty number lines with larger numbers and decimal numbers, as appropriate.

Continue to develop the formal written method for addition with larger numbers (and decimal numbers) and with the addition of three or more numbers:

$$21,848 + 1,523 = 23,371$$

	2	1	8	4	8
+		1	5	2	3
	2	3	3	7	1
		1		1	

Continue to use the language of place value to ensure understanding.

Ensure that the digits that have been 'carried' are recorded under the line in the correct column.

Use the formal written method for the addition of decimal numbers:

$$£154.75 + £233.82 = £388.57$$

	2	3	3	•	8	2
+	1	5	4	•	7	5
	3	8	8	•	5	7
			1			

Ensure that the decimal points line up.

Continue to practise and apply the formal written method throughout Y5, including the addition of more than two numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Five - Subtraction

Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of empty number lines with larger numbers and decimal numbers, as appropriate.

Continue to develop the formal written method for subtraction with three-digit and four-digit numbers (see Y4 guidance), returning to an expanded method and using base ten apparatus, if necessary.

$$503 - 278 = 225$$

		5	0	0	+		0	+	3	
-		2	0	0	+	7	0	+	8	
<hr/>										
		4	0	0	+	9	0	+	13	
-		2	0	0	+	7	0	+	8	
		2	0	0	+	2	0	+	5	=

In this example 503 has to be partitioned into 400+90+13 in order to carry out the subtraction calculation.

Ensure that children are confident in partitioning numbers in this way.

This leads into the formal written method (there is potential for error in this example):

		4	9	13
		5	0	3
-		2	7	8
		2	2	5

There are no tens in the first number (503) so we have to exchange a hundred for 10 tens before we can exchange a ten for ten ones/units.

Consider the use of base ten apparatus to support understanding.

NB It would be appropriate to discuss the use of mental calculation methods with an example like this one, i.e. would an empty number line be a more efficient method for these numbers?

When children are confident, extend with larger numbers and decimal numbers.

$$12,731 - 1,367 = 11,364$$

			6	12	11
	1	2	7	3	1
-		1	3	6	7
	1	1	3	6	4

In this example it has been necessary to exchange from the tens and the hundreds columns.

$$£166.25 - £83.72 = £82.53$$

		16	5	12	
	1	6	6	2	5
-		8	3	7	2
		8	2	5	3

Ensure the decimal points line up.

Continue to practise and apply the formal written method with larger numbers (numbers with more than four-digits) including decimal numbers.

NB If, at any time, children are making significant errors, return to the previous stage in calculation and provide calculations where only one exchange is required.

Multiplication - Year Five

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Build on the work covered in Y4 with the formal method of short multiplication (two-digit or three-digit numbers multiplied by a one-digit number).

$$329 \times 7 = 2,289$$

		3	2	7
×				7
	2	2	8	9
		1	4	

Ensure that the digits 'carried over' are written under the line in the correct column.

$$1,256 \times 4 = 5,024$$

	1	2	5	6
×				4
	5	0	2	4
	1	2	2	

When children are confident introduce multiplication by a two-digit number.

If necessary, return to the grid method and/or expanded method first.

Grid method (two-digit number multiplied by a two-digit number):

$$23 \times 13 = (20 + 3) \times (10 + 3) = 299$$

×	20	3						
10	200	30			2	3	0	
3	60	9		+		6	9	
					2	9	9	

Add the partial products $(200 + 30) + (60 + 9) = 299$

Expanded long multiplication (two-digit number multiplied by a two-digit number):

$$23 \times 13 = 299$$

		2	3						
×		1	3						
			9			(3 × 3)			
		6	0			(3 × 20)			
		3	0			(10 × 3)			
+	2	0	0			(10 × 20)			
	2	9	9						

This leads into...

Compact long multiplication (formal method):

$$23 \times 13 = 299$$

$$\begin{array}{r}
 23 \\
 \times 13 \\
 \hline
 + 69 \quad (3 \times 23) \\
 230 \quad (10 \times 23) \\
 \hline
 299
 \end{array}$$

Use the language of place value to ensure understanding.

Add the partial products.

$$56 \times 27 = 1,512$$

$$\begin{array}{r}
 56 \\
 \times 27 \\
 \hline
 392 \\
 +1120 \\
 \hline
 1512 \\
 \hline
 \end{array}
 \begin{array}{l}
 (7 \times 56) \\
 (20 \times 56)
 \end{array}$$

Use the language of place value to ensure understanding.

In this example there are digits that have been 'carried' over in the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

When children are confident with long multiplication extend with a three-digit number multiplied by a two-digit number:

$$124 \times 26 = 3,224$$

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 7424 \\ + 2480 \\ \hline 3224 \\ 11 \end{array}$$

Use the language of place value to ensure understanding.
Add the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Y6 guidance).

NB If, at any time, children are making significant errors, return to the previous stage in calculation.

Year Five - Division

Divide numbers with up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

NB Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise the formal written method of short division with whole number answers...

$$184 \div 8 = 23$$

$$\begin{array}{r} 23 \\ 8 \overline{) 184} \end{array}$$

Use the language of place value to ensure understanding.

Make the link to the partitioning method (see Y4 guidance).

...and with remainders:

$$432 \div 5 = 86 \text{ r}2$$

$$\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}$$

The remainder can also be expressed as a fraction, $\frac{2}{5}$ (the remainder divided by the divisor):

$$432 \div 5 = 86 \frac{2}{5}$$

Continue to practise, develop and extend the formal method of short division, with and without remainders.

Interpret and express remainders according to the context when solving problems.

NB If, at any time, children are making significant errors, return to the previous stage in calculation.